**Assignment 3**

1) View the following Data Set here. The column on the left is the original input of strings to be sorted or shuffled; the column on the extreme right are the string in sorted order; the other columns are the contents at some intermediate step during one of the 8 algorithms listed below. Match up each algorithm under the corresponding column. Use each algorithm exactly once: (1) Knuth shuffle (2) Selection sort (3) Insertion sort (4) Merge sort(top-down) (5) Merge sort (bottom-up) (6) Quicksort (standard, no shuffle) (7) Quicksort (3-way, no shuffle) (8) Heapsort.

1. Bottom Up Merge Sort

Pairs of four elements are sorted among themselves, similar to what happens in bottom up merge sort.

1. Quick Sort (standard, no shuffle)

First element (navy) becomes the pivot and all elements before ‘navy’ are smaller and all those after ‘navy’ are larger. First step that takes place here is that ‘mist’ and ‘plum’ get swapped.

1. Insertion

Second half of the array is exactly same as the original, because it has not been seen yet.

1. Top Down Merge Sort

First half and the second half are individually sorted and elements are not yet in their final positions.

1. Knuth Shuffle

Elements are placed at random and do not follow a particular logic.

1. Heapsort

The element at position 1 is greater than elements at positions 2 and 3. Element at position 2 is greater than elements at positions 4 and 5 and so on. In general, element at position ‘n’ is greater than elements at positions ‘2n’ and ‘2n + 1’, because it is their parent. This is an example of max-heap.

1. Selection Sort

First half of the array is exactly same as the sorted array, because all the elements in the first half have been selectively placed in their final positions.

1. Quick Sort (3-way, no shuffle)

First element (navy) becomes the pivot and all elements before ‘navy’ are smaller and all those after ‘navy’ are larger. First step that takes place here is that ‘plum’ and ‘palm’ get swapped and plum ends up in the last position

2) Implement Quicksort using median-of-three to determine the partition element. Compare the performance of Quicksort with the Merge sort implementation and dataset from Q1 (HW2). Is there any noticeable difference when you use N=7 as the cut-off to insertion sort? Experiment if there is any value of "cut-off to insertion" at which the performance inverts.

The table below shows the comparative time performance of Median of Three Quick Sort and Quick Sort that reverts to Insertion Sort below a certain length given by cutoff value with that of Top Down Merge Sort. The graph below the table shows the data below plotted against data size.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Data Size** | **Execution Time (ms)** | | | |
| **Top-Down Merge-sort** | **Median of Three Quicksort** | **Median of Three Quicksort (Cutoff=7)** | **Median of Three Quicksort (Cutoff=5)** |
| 1024 | 16 | 7 | 8 | 7 |
| 2048 | 40 | 14 | 17 | 14 |
| 4096 | 91 | 36 | 36 | 35 |
| 8192 | 196 | 79 | 77 | 60 |
| 16384 | 429 | 175 | 174 | 150 |
| 32768 | 1003 | 333 | 348 | 315 |

We see that the time taken for execution increases with data size as expected. The quick sort with median-of -three algorithm out performs merge sort. The order of growth of quick sort is also less than that of merge sort. The time insertion sort takes to sort small sized arrays is less than quick sort. So, cutting off to insertion sort for small sub-arrays pays off as seen from the graph. The trend, however, inverts above the cutoff value of 25.

Hypothesis => for Merge sort

T = a\*Nb

Therefore, 40 = a\*2048b and 91 = a\*4096b

Solving, we get b = 1.186 and a = 0.00473 (approximately)

Thus, **T = 0.00473\*N1.186**

Hypothesis => for Median-of-Three Quick sort

T = a\*Nb

Therefore, 14 = a\*2048b and 36 = a\*4096b

Solving, we get b = 1.345 and a = 0.00049 (approximately)

Thus, **T = 0.00049\*N1.345**

Figure 1 – Performance time comparison of two Quick Sorts with Merge Sort

3) Problem 9.29 From Sedgewick, Algorithms in C++, 3rd Edition: Empirically determine the percentage of time heapsort spends in the construction phase for N=103, 104, 105 and 106.

The table below shows time taken by Heap Sort in construction phase of the algorithm, the total time taken by the algorithm and subsequently, the percentage of time spent in construction phase. Figure 2 plots heap construction time with respect to data size and Figure 3 plots the percent time spent in construction phase against data size. The data size considered in the analysis is 103, 104, 105, and 106.

|  |  |  |  |
| --- | --- | --- | --- |
| **Heap Sort** | | | |
| **Data Size** | **Time in Construction phase** | **Total Time** | **% Time in Construction** |
| 1000 | 1 | 15 | 6.666666667 |
| 10000 | 18 | 215 | 8.372093023 |
| 100000 | 175 | 2546 | 6.873527101 |
| 1000000 | 1457 | 25567 | 5.698752298 |

Figure 2 – Heap Construction time vs Data Size

Figure 3 – Percent time spent in construction phase vs data size

Hypothesis => for Time spent in construction phase by heap sort

T = a\*Nb

Therefore, 18 = a\*10000b and 175 = a\*100000b

Solving, we get b = 0.987 and a = 0.00203 (approximately)

Thus, **T = 0.00203\*N0.987**